



Chinese Society of Aeronautics and Astronautics  
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn  
www.sciencedirect.com



# Reliability design optimization of composite structures based on PSO together with FEA

Chen Jianqiao <sup>a</sup>, Tang Yuanfu <sup>a</sup>, Ge Rui <sup>b</sup>, An Qunli <sup>a,\*</sup>, Guo Xiwei <sup>a</sup>

<sup>a</sup> Department of Mechanics, Huazhong University of Science and Technology, Wuhan 430074, China

<sup>b</sup> Research and Development Center, Wuhan Iron and Steel (Group) Corp., Wuhan 430083, China

Received 13 December 2011; revised 21 February 2012; accepted 12 March 2012

Available online 6 March 2013

## KEYWORDS

Composite structures;  
FEA;  
PSO;  
Reliability analysis;  
Reliability design  
optimization

**Abstract** The present work aims to develop a method for reliability-based optimum design of composite structures. A procedure combining particle swarm optimization (PSO) and finite element analysis (FEA) has been proposed. Numerical examples for the reliability design optimization (RDO) of a laminate and a composite cylindrical shell are worked out to demonstrate the effectiveness of the method. Then a design for composite pressure vessels is studied. The advantages and necessity of RDO over the conventional equi-strength design are addressed. Examples show that the proposed method has good stability and is efficient in dealing with the probabilistic optimal design of composite structures. It may serve as an effective tool to optimize other complicated structures with uncertainties.

© 2013 Production and hosting by Elsevier Ltd. on behalf of CSAA & BUAA.  
Open access under [CC BY-NC-ND license](http://creativecommons.org/licenses/by-nc-nd/3.0/).

## 1. Introduction

Composite materials have been widely applied in construction of components in mechanical, aerospace, shipbuilding, and other industries. Since these composite structures are generally in service under special and severe circumstances, where lots of unavoidable uncertainties exist, it is necessary to take the uncertainties into consideration when designing these structures. In order to accommodate and manage the effects of uncertainties on design

performance, a number of methods such as reliability design optimization (RDO), safety analysis, robust based design, and system identification have been developed.<sup>1–4</sup> The objective of RDO is to seek a design that achieves a targeted probability of failure and ensures expected optimum performance.<sup>5</sup>

In structural reliability analysis, reliability is defined as a multidimensional nonlinear integral. Direct evaluation of such an integral is unfeasible or even impossible in most cases. Therefore, some approximation or simulation methods for probabilistic uncertainty analysis<sup>6–8</sup> have been developed, among which most probable point (MPP) based methods are widely used in RDO since they have the advantages of satisfactory accuracy and moderate computational cost.<sup>9–11</sup> Typical MPP-based methods include the first order reliability method (FORM), the second order reliability method, and the first order saddle point approximation.

A qualified reliability design optimization of composite structures directly depends on two aspects<sup>12,13</sup>: (i) an optimization technique that can efficiently find the global solution; (ii) a

\* Corresponding author. Tel.: +86 27 87543738.

E-mail addresses: [mech-chen@263.net](mailto:mech-chen@263.net) (J. Chen), [huitangyuan@163.com](mailto:huitangyuan@163.com) (Y. Tang), [happygerui@126.com](mailto:happygerui@126.com) (R. Ge), [anqunli@163.com](mailto:anqunli@163.com) (Q. An), [xiwguo@gmail.com](mailto:xiwguo@gmail.com) (X. Guo).

Peer review under responsibility of Editorial Committee of CJA.



Production and hosting by Elsevier

structural analysis tool that can accurately evaluate the stresses of complicated composite structures.

Conventional optimization algorithms are computationally efficient in general, but often encounter a problem that the optimum results are sensitive to the initial condition and differentiation calculations are required. In order to overcome the shortcomings of these algorithms, several biologically inspired evolutionary algorithms<sup>14–18</sup> have been developed. They differ from the most traditional optimization techniques in that they involve a search from a “population” of candidate solutions, not from a single point. As compared with other evolutionary methods, particle swarm optimization (PSO) has no complicated evolutionary operators such as crossover, selection, and mutation. Information in the search procedure of PSO is socially shared among individuals to direct the search towards the best position in the search space.<sup>18</sup> For numerically analyzing composite structures, on the other hand, the commercial FEA code ANSYS<sup>19</sup> provides diversiform element types which can be chosen according to the specific applications of composite structures.

Several researchers have adopted PSO or improved PSO to carry out optimal design of composite structures.<sup>20–25</sup> A technique of applying PSO integrated with general finite element code was developed by Peng et al. to minimize interlaminar normal stresses at the free-edge.<sup>24</sup> Ge et al.<sup>25</sup> proposed an improved PSO algorithm and applied it to RDO of composite materials. However, classic lamination theory was used to obtain stresses. Up to now, few works have been reported in literature about the use of evolution algorithms (EAs) together with FEA for RDO of composite structures.

In this paper, a design procedure for RDO is proposed which is based on PSO together with the FEA code ANSYS. The approach includes three modules: (1) stress analysis by ANSYS; (2) reliability calculation; and (3) optimization. Numerical examples for reliability-based optimum design of laminates, composite cylindrical shells, and composite pressure vessels are worked out to demonstrate the effectiveness of the proposed method.

## 2. Reliability analysis for composites and reliability based design procedure

### 2.1. Tsai–Wu failure criterion

Tsai–Wu failure criterion<sup>26</sup> is the most reasonable failure criterion for composites. It can be expressed as

$$FI = F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + 2F_{12}\sigma_1\sigma_2 + 2F_{23}\sigma_2\sigma_3 + 2F_{31}\sigma_3\sigma_1 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 \leq 1 \quad (1)$$

where

$$F_1 = 1/X_T - 1/X_C, \quad F_2 = 1/Y_T - 1/Y_C,$$

$$F_3 = 1/Z_T - 1/Z_C$$

$$F_{11} = 1/(X_TX_C), \quad F_{22} = 1/(Y_TY_C), \quad F_{33} = 1/(Z_TZ_C),$$

$$F_{44} = 1/S_{yz}^2, \quad F_{55} = 1/S_{zx}^2, \quad F_{66} = 1/S_{xy}^2$$

$$F_{12} = (-1/2)\sqrt{F_{11}F_{22}}, \quad F_{23} = (-1/2)\sqrt{F_{22}F_{33}},$$

$$F_{31} = (-1/2)\sqrt{F_{33}F_{11}}$$

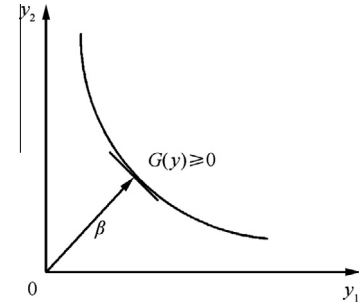


Fig. 1 Geometrical illustration of the reliability index  $\beta$ .

where  $X_T$  and  $X_C$  are tensile and compressive strength in the longitudinal direction, respectively;  $Y_T$ ,  $Z_T$ ,  $Y_C$  and  $Z_C$  are tensile and compressive strength in the transversely isotropic surface, respectively;  $S_{xy}$ ,  $S_{yz}$  and  $S_{zx}$  are shear strength in the transversely isotropic surface. Therefore, the limit state function in terms of Tsai–Wu failure criterion can be expressed

$$G = 1 - FI \quad (2)$$

The composite element is in operating state if  $G > 0$ , in failure state if  $G < 0$ , and in limit state if  $G = 0$ .

### 2.2. Reliability analysis

In the structural reliability analysis, the reliability is defined as

$$R_s = P[g(\mathbf{d}, \mathbf{X}) \geq 0] = \int_{g(\mathbf{d}, \mathbf{x}) \geq 0} f_X(\mathbf{x}) d\mathbf{x} \quad (3)$$

where  $g$  is a performance function or the limit state function,  $\mathbf{d}$  a vector consisted of the design variables, and  $f_X$  the joint probability density function (PDF) of the random variables  $\mathbf{X}$ .

The FORM is used for calculating the structural reliability. In the method, the random variable vector  $\mathbf{X}$  is firstly transformed to  $\mathbf{Y}$ , the vector of equivalent uncorrelated standard normal variables.<sup>27–29</sup> The component reliability index is computed as  $\beta = (\mathbf{y}^{*T}, \mathbf{y}^*)^{1/2}$  in which  $\mathbf{y}^* = [y_1 \ y_2]$  is the point in the limit state  $G(\mathbf{y}) = 0$  at a minimum distance from the origin and is referred to the most probable point (MPP) to failure. A geometrical illustration for a limit state involving two random variables,  $y_1$  and  $y_2$ , is shown in Fig. 1. The failure probability is approximated by

$$P(G \geq 0) = \Phi(\beta) \quad (4)$$

where  $\Phi$  is the cumulative distribution function of the standard normal variable.

The MPP can be found by either an iterative calculation or solving a minimization problem:

$$\mathbf{y}_{i+1} = [\mathbf{y}_i^T \alpha_i - G(\mathbf{y}_i)/|\nabla G(\mathbf{y}_i)|] \alpha_i \quad (5)$$

$$\min_{\mathbf{y}} \|\mathbf{y}\|$$

$$\text{s.t. } G(\mathbf{y}) = 0 \quad (6)$$

where  $\nabla G(\mathbf{y}_i)$  is the gradient vector of the limit state function at  $\mathbf{y}_i$ , and  $\alpha_i$  the unit vector normal to the limit state surface away from the origin.

### 2.3. PSO

PSO which is an evolutionary global algorithm has gained popularity recently.<sup>17,18</sup> Similar to other existing EAs, PSO is a population-based optimization method. Distinct from other EAs where knowledge is destroyed between generations, individuals in the population of PSO retain memory of known good solutions as the search for better solutions continues. Hence, PSO has higher speed of convergence than other evolutionary search algorithms. The other advantage of PSO is that it's easy to implement and there are fewer parameters to adjust. The velocity vector of each particle is calculated according to the formula:

$$\mathbf{v}_k^i = w\mathbf{v}_{k-1}^i + c_1r_1(\mathbf{p}_{k-1}^i - \mathbf{x}_{k-1}^i) + c_2r_2(\mathbf{p}_{k-1}^g - \mathbf{x}_{k-1}^i) \quad (7)$$

where the superscript  $i$  denotes the particle and the subscript  $k$  the iteration number;  $\mathbf{v}$  denotes the velocity and  $\mathbf{x}$  the position;  $r_1$  and  $r_2$  are uniformly distributed random numbers in the interval  $[0, 1]$ ;  $c_1$  and  $c_2$  are the acceleration constants;  $w$  is the inertia weight;  $\mathbf{p}_{k-1}^i$  is the best position of particle  $i$  and  $\mathbf{p}_{k-1}^g$  the global best position attained by the swarm at iteration  $k-1$ . The position of each particle at iteration  $k$  is calculated using the formula:

$$\mathbf{x}_k^i = \mathbf{x}_{k-1}^i + \mathbf{v}_k^i \quad (8)$$

### 2.4. Procedure for the reliability-based optimum design of structures

The optimization procedure for the reliability-based optimum design can be described in Fig. 2. The approach includes three modules: (1) stress analysis by ANSYS; (2) reliability calculation; and (3) optimization. Apart from ANSYS, the procedure is essentially written in the compute software MATLAB.

Firstly, the particles are initialized with random position values and random velocities. Secondly, ANSYS is called at background to calculate the stress of the structure, and outputs these data to an external file. Thirdly, MATLAB reads the file and calculates the system reliability. The optimization algorithm evaluates fitness of each swarm and then accounts the best previous experience and the best experience of all other members of the swarm at current iteration. Each swarm updates itself through the best solutions mentioned above. The updated swarms are returned to ANSYS for the next iteration. This process is repeated until the number of iteration reaches the pre-determined maximum iteration number. The data from

ANSYS and MATLAB are exchanged, back and forth, with each other in the whole optimization process.

### 3. Reliability-based design of fiber reinforced plastics (FRP) laminates and composite cylindrical shells

#### 3.1. Optimization design of an FRP laminate and the convergence validation of PSO–FEA procedure

When FRP laminated plates are used in aerospace structures, the primary concern to designers is how to reduce their weight without compromising their performance. Such issues can usually be summarized as follows: minimize the structural weight with reliability constraints. The design model can be formulated as

$$\begin{aligned} \min_{\mathbf{d}} \quad & h(\mathbf{d}) = \sum_{i=1}^n h_i(\mathbf{d}) \\ \text{s.t.} \quad & P\{g \geq 0\} = \Phi(\beta) \geq R_t \end{aligned} \quad (9)$$

where  $\mathbf{d}$  is the design variable (e.g., ply angle, etc.),  $h(\mathbf{d})$  the total ply thickness,  $\Phi(\beta)$  the system reliability, and  $R_t$  the target system reliability.

Consider a simply supported symmetric laminated composite plate ( $20 \text{ cm} \times 12.5 \text{ cm}$ ) under both compression load  $N_x = 500 \text{ kN/m}$  and uniform transverse load  $p = 0.2 \text{ MPa}$ , as shown in Fig. 3.

The stacking structure is  $[0/+45/-45/90]_s$ . The thicknesses of  $0^\circ$  layer and  $90^\circ$  layer are both  $0.25 \text{ mm}$ , and those of  $\pm 45^\circ$  layer and  $-45^\circ$  layer are  $h_1 \times 0.125 \text{ mm}$ ,  $h_2 \times 0.125 \text{ mm}$ , respectively. The laminate is made of T300/5208 graphite/epoxy material with  $E_1 = 181 \text{ GPa}$ ,  $E_2 = 10.3 \text{ GPa}$ ,  $G_{12} = 7.17 \text{ GPa}$ , and  $\mu_{12} = 0.28$ . The strength parameters are considered as normally distributed random variables of independence and their distribution characteristics are shown in Table 1. The objective function of this problem is  $f = h_1 + h_2$  with  $h_1$  and  $h_2$  being the design variables. The target system reliability is  $R_t = 0.99$ .

The proposed method combining PSO and ANSYS is used in the reliability-based optimum design calculation. The population size and the maximum iteration are 20 and 40, respectively. The results are listed in Table 2. Since performing finite element analysis for composite structure is a time-consuming process, the computational cost required by the reliability calculation and optimization is negligibly low, as compared to that by finite element analysis. Thus, approximately, the efficiency of the two methods is compared by the number of finite element analyses ( $N_{\text{FEA}}$ ). It is observed that the method combining PSO and ANSYS obtains a better solution than that obtained by the optimum toolbox of ANSYS.

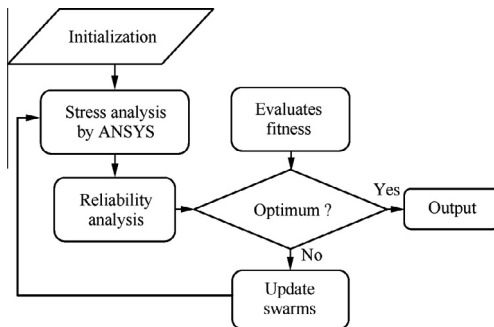


Fig. 2 Procedure for the reliability-based optimum design.

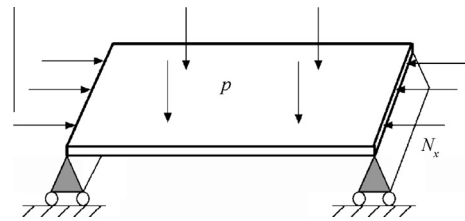


Fig. 3 A laminate subjected to axial compression load and uniform lateral load.

**Table 1** Random variables.

Distribution	$X_T$ (MPa)	$X_C$ (MPa)	$Y_T$ (MPa)	$Y_C$ (MPa)	$S$ (MPa)
Mean	1500	1500	48	246	68
Standard deviation	150	150	4.8	24.6	6.8

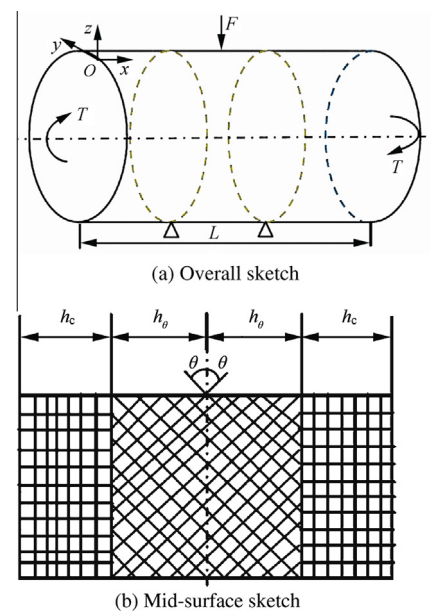
**Table 2** Reliability-based design results.

Method	Initial solution		Optimal solution		$f$ ( $10^{-4}$ m)	$\Phi(\beta)$	$N_{FEA}$
PSO-ANSYS	Random		5.013	4.711	9.724	0.990	800
ANSYS	2.000	2.000	6.813	7.039	13.852	0.998	626
	4.000	4.000	5.116	4.952	10.069	0.992	511
	6.000	6.000	5.235	5.018	10.253	0.993	478
	8.000	8.000	6.478	6.320	12.798	0.996	557

Meanwhile, the proposed procedure can reach the global optimum regardless of randomly generated initial candidate solutions (population). The quality of the optimal solution of the optimum toolbox of ANSYS which is based on the gradient optimization theory and function approximation, however, depends on the selection of an initial solution: a better initial solution leads to a better optimal solution, and a worse one reduces the quality of the optimal solution. However, the initial solution in the PSO-ANSYS procedure is randomly generated, and the quality of the optimal solution has nothing to do with the selection of the initial solution. Due to employment of the function approximations, the computational cost required by the optimum toolbox of ANSYS is less than that by the proposed method as shown in Table 2. Thus, our next work will focus on improving the efficiency of the proposed method.

### 3.2. Optimization design of a fiber-reinforced composite cylindrical skirt

Consider a fiber-reinforced composite cylindrical skirt for solid rocket cases as shown in Fig. 4,<sup>30</sup> in which  $h_\theta$  and  $h_c$  stand for the thicknesses of the inner (angle-ply layer) and outer (cross-ply layer) region. Its length  $L = 0.5$  m, radius  $R = 0.3$  m (from the center to the mid-surface of the composite layers), and total thickness  $h$  is considered as the objective. Let the mid-surface of the cylindrical shell of the skirt be the reference surface, and let the origin of the coordinates be located at one end of the cylinder. The fiber orientations are symmetric with respect to the mid-surface of the cylindrical shell. The orthogonal coordinates  $x$ ,  $y$ , and  $z$  are measured in the longitudinal, circumferential, and radial directions, respectively. The shell has a symmetric and balanced composite structure with a stacking sequence  $[90/0/+ \theta/-\theta]_s$ .  $\theta$  is the angle between the fiber direction and the longitudinal axis. The composite cylindrical skirt with simply supported boundary conditions

**Fig. 4** A fiber-reinforced composite cylindrical shell.

( $w = v = 0$  at  $x = L/5, 4L/5$ ) is subjected to torque  $T$  at the ends and a concentrated force  $F$  at the middle.

The material properties are given as  $E_x = 137$  GPa,  $E_y = 8.17$  GPa,  $G_{xy} = 4.75$  GPa, and  $\nu_{xy} = 0.316$ . The strength parameters are normally distributed random variables (see Table 3).

The design objective is to determine the thicknesses  $h_\theta$ ,  $h_c$  and the angle  $\theta$ , to minimize the total weight or equivalently, the total thickness of the shell under the probabilistic constraint  $R \geq 0.9938$ . The constraint condition is equivalent to:

**Table 3** Statistical properties of strength parameters.

Distribution	$X_T$ (MPa)	$X_C$ (MPa)	$Y_T$ (MPa)	$Y_C$ (MPa)	$S$ (MPa)
Mean	1470	980	39.2	78.4	78.4
Standard deviation	147	98	3.92	7.84	78.4

**Table 4** Optimal design results under different values of  $F$ .

$F$ (kN)	$T$ (kN·m)	$h_0$ (mm)	$h_c$ (mm)	$\theta$ (°)	$f$ (mm)	$\beta$
10	70	5.06	4.08	33.71	9.14	2.501
20	70	8.17	5.16	38.08	13.33	2.502
50	70	13.17	8.96	35.22	22.13	2.501
70	70	16.72	9.28	34.48	26.00	2.501

reliability index  $\beta \geq 2.5$ . The optimization design problem can be modeled as

$$\begin{aligned}
 &\text{Find } h_0, h_c, \theta \\
 &\text{Min } f(h) = h_0 + h_c \\
 &\text{s.t. } P(g \geq 0) \geq 0.9938 \text{ or } \beta \geq 2.5 \\
 &\quad 0 \leq h_0 \leq R/10, \quad 0 \leq h_c \leq R/10 \\
 &\quad 0 \leq h_c + h_0 \leq R/10, \quad 0^\circ \leq \theta \leq 90^\circ
 \end{aligned} \quad (10)$$

The proposed method combining PSO and ANSYS is adopted in the optimum design. The optimal results for the composite cylindrical shell under different combinations of concentrated force  $F$  and fixed torque  $T$  are listed in Table 4. As  $F$  increases, both the cross-ply and angle-ply layer thickness increase. But the latter (inner layer) is always thicker than the former (outer layer). The optimal fiber angles vary in a small range (33–38°).

Table 5 shows the optimal design results for a shell with  $[\theta/-\theta/90/0]_s$ . As compared with Table 4, similar features are observed, i.e., the cross-ply layer (inner layer) is thicker than the angle-ply one (outer layer) except for  $F = 20$  kN. The optimal fiber angles are greater than those for  $[90/0/+\theta/-\theta]_s$ . The total thickness displays a similar change tendency as in Table 4.

For every load condition, the total thickness is greater than that of  $[90/0/+\theta/-\theta]_s$ , indicating that  $[90/0/+\theta/-\theta]_s$  is better than  $[\theta/-\theta/90/0]_s$  for the loading conditions.

#### 4. Reliability-based design optimization of a composite pressure vessel

##### 4.1. Conventional strength design for composite pressure vessels

Filament-wound structures such as pressure vessels, pipes and motor cases of rockets are widely used in aerospace applications.<sup>31,32</sup> In this section, the reliability-based optimal design of a composite pressure vessel is investigated.

A composite pressure vessel is composed of a cylindrical part and a head part. In the cylindrical region, there are inner helical composite layers and outer circular layers. The outer circular layers are assigned at a fixed fiber angle ( $\theta \approx 80$ – $90^\circ$ ) and a certain thickness  $t_0$ . If the wrap angle of the helical layer

in the cylindrical part is  $\alpha_0$  and the corresponding thickness is  $t_0$ , then the fiber angle and thickness in the head part are determined according to following relations<sup>29</sup>:

$$\begin{cases} \sin \alpha = \frac{R}{x} \sin \alpha_0 \\ t_x = \frac{R}{x} \cdot \frac{\cos \alpha_0}{\cos \alpha} t_0 \end{cases} \quad (11)$$

where  $R$ ,  $\alpha_0$ ,  $t_0$  are the radius, fiber angle, and thickness of the helical layer in the cylindrical region, while  $x$ ,  $\alpha$ ,  $t_x$  are the radius, fiber angle, and thickness of any point at the head shape.

In the conventional strength design for composite pressure vessels, only strength of the cylindrical part is considered. The design results based on the equi-strength criterion are

$$\begin{cases} t_0 = \frac{pR}{2[\sigma] \cos^2 \alpha_0} \\ t_\theta = \frac{3pR}{2[\sigma]} - t_0 \end{cases} \quad (12)$$

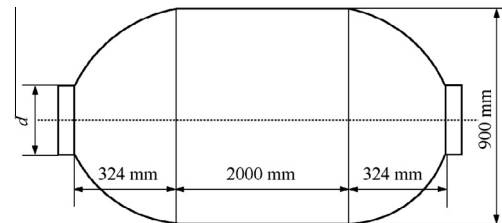
where  $[\sigma]$  denotes the allowable stress in the fiber direction.

Consider a composite pressure vessel under internal pressure loads  $p$ . The cylindrical region of the vessel is composed of an inner helical layer and an outer circular layer. The geometry of the vessel is shown in Fig. 5. The wrap angle of the helical layer in the cylindrical portion is  $\pm\alpha_0$ , and the corresponding thickness is  $h_1$ . The circular layer thickness is  $h_2$ . The fiber angle and thickness in the head part are determined according to Eq. (12). The composite material is T800/Epoxy (Toray). Mechanical properties are listed in Table 6. Table 7 shows the statistical characteristics of the strength parameters.  $p = 10$  MPa,  $\alpha_0 = \pm 20^\circ$ ,  $\theta = 90^\circ$ , and  $[\sigma] = 767.7$  MPa. The design results are:  $t_0 = 5.47$  mm,  $t_\theta = 3.32$  mm.

FE analysis based on ANSYS is conducted for the vessel. Figs. 6 and 7 show the fiber direction stress and the transverse stress in the most inner layer (helical layer). The maximum stresses locate at the transitional part due to stress concentration. The fiber direction stress satisfies the strength condition, i.e.,  $\sigma_{LL} < [\sigma] = 767.7$  MPa. The maximum transverse stress is  $\sigma_{TT} = 33.39$  MPa, which approaches the strength limit (36.4 MPa). Considering the variation of composite strength, it can be concluded that the above design can be taken as a reference but has the risk of failure. Alternative design methods are needed to get more reliable solutions that are described in the following section.

**Table 5** Optimal design results for the shell with stacking sequence  $[\theta/-\theta/90/0]_s$ .

$F$ (kN)	$T$ (kN·m)	$h_0$ (mm)	$h_c$ (mm)	$\theta$ (°)	$f$ (mm)	$\beta$
10	70	4.10	6.00	39.11	10.10	2.504
20	70	8.09	6.66	43.60	14.75	2.508
50	70	10.51	13.83	41.42	24.34	2.501
70	70	13.18	16.04	44.71	29.22	2.518

**Fig. 5** Geometry of the composite pressure vessel.

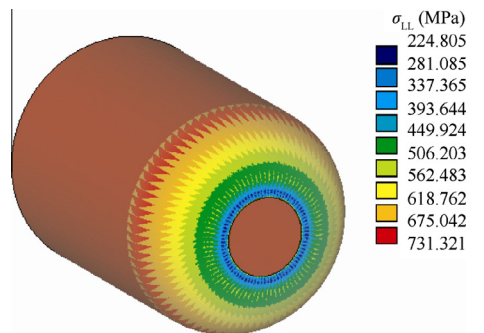
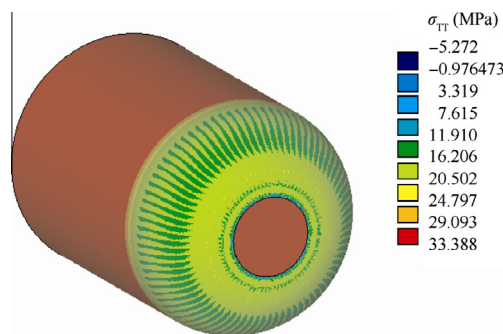


**Table 6** Material properties of T-800/Epoxy.

$E_2$ (GPa)	$E_2, E_3$ (GPa)	$G_{12}, G_{13}$ (GPa)	$G_{23}$ (GPa)	$\nu_{12}, \nu_{13}$	$\nu_{23}$
142	3.14	4.69	1.0	0.33	0.45

**Table 7** Strength parameters.

Distribution	$X_T$ (MPa)	$X_C$ (MPa)	$Y_T, Z_T$ (MPa)	$Y_C, Z_C$ (MPa)	$R, S, T$ (MPa)
Mean	2687	1441	36.4	70	59.6
Standard deviation	268.7	144.1	3.64	7.0	5.96

**Fig. 6** Direct stress  $\sigma_{LL}$  in the fiber direction in the inner layer.**Fig. 7** Direct stress  $\sigma_{TT}$  in the transverse direction in the inner layer.

#### 4.2. Reliability design for composite pressure vessels

To account for the effect of uncertainties in the design of pressure vessels, a reliability design is conducted. In the above example, all the random variables are assumed to be uncorre-

**Table 8** Optimal solutions under different internal pressures.

$P$ (MPa)	$h_1$ (mm)	$h_2$ (mm)	$\alpha_0$ (°)	$H$ (mm)
5	4.21	2.59	10.13	11.01
10	8.55	2.63	18.18	19.73
15	9.59	4.07	20.58	23.25
20	11.61	5.46	21.80	28.68
30	15.62	7.62	26.55	38.86
40	19.37	10.28	28.06	49.02
50	22.56	11.71	28.93	56.82

lated normally distributed variables for the sake of illustration. The optimization problem is expressed as

$$\begin{aligned} &\text{Find } \alpha_0, h_1, h_2 \\ &\text{Min } H = 2h_1 + h_2 \\ &\text{s.t. } \Phi(\beta_s) \geq 0.99, \quad 10 \leq \alpha_0 \leq 85 \end{aligned} \quad (13)$$

Using the method combining PSO and ANSYS, the optimal laminate structure corresponding to different internal pressure loads are computed and listed in Table 8.

It can be seen that the thickness of the optimum structure increases with the increase of the internal pressure as expected. It is worthy to mention that the wrap angle ( $\alpha_0$ ) of the helical layer in the cylindrical region also increases as the load increases. This is because that the rate of the circumferential forces increases faster than that of the meridional stress when the internal pressure increases. The helical layer bears not only the meridional stress, but also a part of the circumferential forces. To make the stress distribute more reasonably, the wrap angle ( $\alpha_0$ ) of the helical layer should increase as the load increases.

#### 5. Conclusions

Owing to the high sensitivity of strength to load conditions and other factors for composite structures, high reliability is usually required for them. The present work concentrates on developing a method for the reliability-based optimum design of composite structures. The method includes three modules: (1) stress analysis; (2) reliability calculation; and (3) optimization. ANSYS is employed to accurately evaluate stresses of complicated composite structures, and PSO is used to find the global solution. Except stress analysis, the proposed method is written in MATLAB. The data from ANSYS and MATLAB are exchanged in each cycle. Several examples are carried out to demonstrate the effectiveness of the proposed method. For the reliability-based design optimization of composite pressure vessels, we get the change law of wrap angle  $\alpha_0$  and layer thickness with the internal pressure load  $p$ , which provides the engineering application with very important reference values. Our future work will focus on reducing the computational cost of the method.

#### Acknowledgements

This study was supported by National Natural Science Foundation of China (No. 10772070) and National Basic Research Program of China (No. 2011CB013800).

## References

- Schuëller GI, Jensen HA. Computational methods in optimization considering uncertainties—an overview. *Comput Methods Appl Mech Engrg* 2008;**198**(1):2–13.
- Ge R, Chen JQ. RBD of composites under the mixed uncertainties and the optimization algorithm. *Acta Mechanica Solida Sinica* 2008;**21**(1):19–27.
- Chen JQ, Wei JH, Xu YR. Fuzzy reliability analysis of laminated composites. *Struct Eng Mech* 2006;**22**(2):665–83.
- Doltsinis I, Kang Z. Robust design of structures using optimization methods. *Comput Methods Appl Mech Engrg* 2004;**193**(22–):2221–37.
- Melchers RE. *Structural reliability analysis and prediction*. Chichester: Wiley; 1999.
- Olsson A, Sandberg G, Dahlblom O. On Latin hypercube sampling for structural reliability analysis. *Struct Saf* 2003;**25**(1):47–68.
- Sakamoto J, Mori Y, Sekioka T. Probability analysis method using fast Fourier transform and its application. *Struct Saf* 1997;**19**(1):21–36.
- Huang BQ, Du XP. Probabilistic uncertainty analysis by mean-value first order saddlepoint approximation. *Reliab Eng Syst Saf* 2008;**93**(2):325–36.
- Hohenbichler M, Gollwitzer S, Kruse W, Rackwitz R. New light on first- and second-order reliability methods. *Struct Saf* 1987;**4**(4):267–84.
- Du XP, Sudjianto A. First order saddlepoint approximation for reliability analysis. *AIAA J* 2004;**42**(6):1199–207.
- Jian T, Kyung KC, Young HP. Design potential method for robust system parameter design. *AIAA J* 2001;**39**(4):667–77.
- Rao SS. *Engineering optimization*. 3rd ed. New York: John Wiley; 1996.
- Carter ADS. *Mechanical reliability and design*. New York: Wiley; 1997.
- Haupt RL, Haupt SE. *Practical genetic algorithms*. New York: Wiley; 1988.
- Peterson C, Söderberg B. *Artificial neural networks*. New York: Wiley; 1993.
- Thomas B. *Evolutionary algorithms in theory and practice: evolution strategies, evolutionary programming, genetic algorithms*. UK: Oxford University Press; 1996.
- Kennedy J, Eberhart R. *Swarm intelligence*. San Diego: Academic Press; 2001.
- Kennedy J, Eberhart R. Particle swarm optimization. In: *Proceeding of the IEEE international conference on neural networks*. 1995; p. 1942–8.
- ANSYS Inc. *ANSYS coupled-field analysis guide release 5*. 3rd ed. SAP: IP Inc.; 1997.
- Suresh S, Sujit P, Rao A. Particle swarm optimization approach for multi-objective composite box-beam design. *Compos Struct* 2007;**81**(4):598–605.
- Tang YF, Chen JQ, Peng WJ. Probabilistic optimization of laminated composites considering both ply failure and delamination based on PSO and FEM. *Tsinghua Sci Technol* 2009;**14**(S2):89–93.
- Chang N, Yang W, Zhao F. Layering parameters design of laminated wing covering board based on particle swarm algorithm. *J Mach Des* 2008;**25**(11):53–6 [Chinese].
- He LT, Wan XP, Zhao MY. Optimization design on the layout of composite material aerofoil structure based on the improved particle swarm algorithm. *J Mach Des* 2008;**25**(5):9–11 [Chinese].
- Peng WJ, Chen JQ, Gu MK, Tu WQ. A particle swarm optimization algorithm for minimizing interlaminar normal stresses at the free-edge of composite laminate. *Mech Sci Technol Aerosp Eng* 2009;**28**(11):1496–500 [Chinese].
- Ge R, Chen JQ, Wei JH. Reliability optimal design of composite materials based on the improved particle swarm optimization algorithm. *Mech Sci Technol* 2007;**26**(2):257–60 [Chinese].
- Tsai SW, Wu EM. A general theory of strength for anisotropic materials. *J Compos Mater* 1971;**5**:58–80.
- Haldar A, Mahadevan S. *Probability, reliability and statistical methods in engineering design*. New York: John Wiley & Sons; 2000.
- Kamand TY, Chan ES. Reliability formulation for composite laminates subjected to first-ply failure. *Compos Struct* 1997;**38**(1–4):447–52.
- Miki M. Reliability-based optimization of fibrous laminated composites. *Reliab Eng Syst Saf* 1997;**56**(3):285–90.
- Liang CC, Chen HW. Optimum design of fiber-reinforced composite cylindrical skirts for solid rocket cases subjected to buckling and overstressing constraints. *Compos B Eng* 2003;**34**(3):273–84.
- Kabir MZ. Finite element analysis of composite pressure vessels with a load sharing metallic liner. *Compos Struct* 2000;**49**(3):247–55.
- Park JS, Kim CU, Kang HK, Hong CS, Kim CG. Structural analysis and strain monitoring of the filament wound motor case. *J Compos Mater* 2002;**36**(20):2373–88.

**Chen Jianqiao** is a professor and Ph.D. advisor in the Department of Mechanics at Huazhong University of Science and Technology. He received his Ph.D. degree from Nagoya University. His research interests include: strength and reliability analysis of engineering materials, fracture mechanics and structural integrity, multidisciplinary design optimization.

**Tang Yuanfu** is a Ph.D. student in the Department of Mechanics at Huazhong University of Science and Technology. His area of research includes optimization algorithms and optimization under uncertainty.

**Ge Rui** received his Ph.D. from Huazhong University of Science and Technology in 2008, and then worked in Wuhan Iron and Steel (Group) Corp. His main research interests are optimization algorithm and reliability-based design optimization of composite materials.

**An Qunli** received his Ph.D. from Huazhong University of Science and Technology in 2001, and then became a teacher there. His main research interests are optimization design methods and high performance computing methods in composite material field.

**Guo Xiwei** is a Ph.D. student at Huazhong University of Science and Technology. His main research area covers pedestrian evacuation dynamics and complex structure modeling.